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14. ABSTRACT

We have developed new techniques for solving matrix optimization problems, such as bilinear matrix inequalities and matrix rank minimization problems. These techniques have enabled us to developed new extensions of Lyapunov theory, which allowed us to analyze the stability and performance of a wide variety of complex systems that could not be handled before. This includes systems with mode-switching logic, hysteresis and saturation nonlinearities and asynchronous clocks. We have also developed optimization-based frameworks for the simultaneous probing and control of uncertain systems, as well as for simultaneous control and communication resource allocation. We have also developed new tools for robust control which compute optimal uncertainty models directly from frequency domain data, and compute reduced order controllers with guaranteed stability properties.

15. SUBJECT TERMS

bilinear matrix inequality (BMI), rank minimization, Lyapunov theory, hybrid systems, asynchronous systems, hysteresis, saturation, dual control, networked control, uncertainty model, reduced order controller

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METHODS FOR MATRIX OPTIMIZATION PROBLEMS IN CONTROL

Final Report

AFOSR F49620-98-1-0147

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Contents

1 Objectives

The objectives of our research were to develop new techniques for solving matrix optimization problems such as linear matrix inequalities (LMIs) and bilinear matrix inequalities (BMIs), and to apply these techniques to the analysis and control of complex dynamical systems.

To a large extent, these objectives have been successfully achieved. We have developed effective new methods for solving BMI's and related matrix optimization problems known as rank minimization problems. We have also developed new extensions of Lyapunov theory, which allow us to analyze the stability and performance of complex systems that could not be handled before. Furthermore, a number of surprising new results have been obtained in the process.

2 Accomplishments

2.1 New techniques for solving matrix optimization problems

2.1.1 Homotopy method for solving bilinear matrix inequalities (BMIs)

On the subject of methods for solving *bilinear matrix inequality* (BMI) problems we have proposed a path-following (homotopy) method for (locally) solving such problems. The idea is to linearize the BMI at each step and compute a perturbation to the control gains using semidefinite programming (SDP) that slightly improves the closed-loop performance. In other words, the BMI is solved by solving a sequence of LMIs along a path parameterized by the closed-loop system performance. The result of this research has been reported in the conference paper [HHB99b]. The effectiveness of the approach has been demonstrated on several practical examples such as simultaneous sensor/actuator placement and controller design, simultaneous stabilization, $\mathcal{H}_2/\mathcal{H}_\infty$ controller design, etc.

2.1.2 Rank minimization

Problems involving *minimizing the rank of a matrix* subject to convex constraints on the matrix arise in a variety of fields such as control systems analysis and design, system identification, statistical signal processing and psychometrics. In the specific context of control, it is generally the case that optimal controller design techniques, such as LQG or H_∞ synthesis, tend to produce high order controllers (order of plant plus noise plus uncertainty and performance weights). This high controller order is one of the biggest stumbling blocks in implementing an optimal controller in practice, and it is well known that the problem of rank minimization is intimately connected to the problem of low order controller synthesis.

Except in some special cases, solving the rank minimization problem (globally) is very difficult. When the matrix variable is symmetric and positive semidefinite, one simple and surprisingly effective heuristic is to minimize its trace in place of its rank . This results in a semidefinite program (SDP), which can be efficiently solved. Another heuristic is to minimize the logarithm of the determinant of the matrix in place of its rank, using a local optimization method.

We have shown how to extend these two heuristic methods to the general case of nonsquare matrices. The extension of the trace heuristic turns out to be to minimize the sum of the singular values of the matrix, which can be done via SDP.

We have also provided some theoretical support for the use of the trace heuristic and its extension to general matrices, by showing that the sum of singular values is in fact the convex envelope of the rank objective over the set of matrices with a given norm bound. Further, we give a version of the log-determinant heuristic and show that it refines the result of the trace method. We also showed that the trace and log-determinant heuristics reduce to known heuristic methods for finding sparse (vector) solutions of convex optimization problems .

We have demonstrated the methods on several example problems, including minimum order system approximation, Hankel matrix rank minimization, low-order H_∞ controller design, the Frisch problem, and Euclidean distance matrix problems. This research is reported in [FHB01a, FHB01b, Faz02].

2.1.3 Parser for convex optimization

We have developed a *new algorithm for automatic parsing* (and solving) of convex optimization problems described in a generic algebraic language. The parser/solver recognizes a reasonable fraction of nonlinear convex optimization problems, and will greatly reduce the two principal barriers to practical use of convex optimization: recognizing convex problems, and implementation of a custom solver.

A proof of principle version of this parser has been successfully implemented and presented in [Cru01]. The final version will be reported in the Ph.D. thesis of the same author.

2.2 Lyapunov-based methods for complex and nonlinear dynamical systems

2.2.1 Hybrid dynamical systems

Significant progress was made in the area of analysis and control of *hybrid dynamical systems*. These are systems that can be used to model control systems which have logic-based modes of operation, and hence have discrete as well as continuous state dynamics. Examples of such systems include timing circuits, process control, computer disk drives, multi-mode systems. These systems do not fall within the framework of control theory, and therefore, a new theory and control methodology is required for analyzing such practically important systems. These systems can exhibit very complex behavior.

We have proposed a class of Lyapunov functionals for analyzing such systems. This class is more general than piecewise-quadratic Lyapunov functions that were previously proposed by us and other researchers [HB98c, RJ97, JR98] for the study of piecewise-linear systems. This class can be thought of as a generalization of the Lyapunov functional proposed by Yakubovich for systems with hysteresis nonlinearities [Yak65] which incorporate path-integral terms that take into account the energy dissipation mechanism of passive hysteresis nonlinearities. Moreover, searching over this class of Lyapunov functionals to prove different specifications for a given hybrid system (e.g., stability) can be cast as semidefinite

programs (SDPs), which can then be solved efficiently using widely available software [VB94, FK95, WB96, GN93, AHNO97, Bor97]. These results are described briefly in [HBH99b] and extensively in the thesis [Has01], where the effectiveness of the methods have been demonstrated on a number of practical examples.

2.2.2 Asynchronous systems

We have also investigated Lyapunov-based methods in the analysis of *asynchronous dynamical systems*. These are continuous systems driven by discrete events such as control systems over asynchronous packet-switched networks, distributed parallelized algorithms, and queuing networks. We have proposed a Lyapunov-based methodology using SDP for analyzing asynchronous systems. A paper has already been submitted to the 1999 Conference on Decision and Control [HBH99a]. The method is effective as demonstrated by several practical examples.

2.2.3 Linear systems with saturation and hysteresis

Research on the analysis of systems with *multiple saturation* and *multiple hysteresis nonlinearities* in robust control has resulted in the conference papers [HB98e, HB98d, PHH99]. We have demonstrated that SDP can be used very effectively in analyzing such systems.

2.3 Matrix optimization for low-authority and dual control

2.3.1 Low-authority controller design

We also finished work in the area of *low-authority controller design*. The premise in low-authority control (LAC) is that the actuators have limited authority, and so cannot greatly modify the system dynamics. The main use of LAC is in lightly damped large structures with many elastic modes, where LAC is used to provide a small amount of damping in a wide range of modes for maximum robustness. Our LAC design method is based on convex programming (LP, SDP, SOCP)

and can therefore deal with very large-scale problems. It is possible to formulate many different design specifications such as eigenvalue-placement, robustness, disturbance rejection, limits on feedback gains, static structural constraints, etc. Our method gives a powerful heuristic for solving the actuator/sensor selection and controller topology design problem via ℓ_1 norm minimization. A journal paper is due to appear in the AIAA Journal on Guidance, Control, and Navigation [HHB99a]. A conference paper giving preliminary results has already been published [HHB98a].

2.3.2 Dual control

Research was also pursued in the area of *optimization-based control*, in particular for the *dual control problem*. With the currently available convex optimization techniques, and given ever increasing processor speed and memory, convex programs can be solved in real-time at ever faster rates, which opens the way to many new control policies. We investigated the problem of controlling uncertain systems, where a policy for joint identification and control (or *dual control*) is required. The first results of this work led to a research paper [LB99], presented at the 1999 American Control Conference, and a journal paper is in progress. This paper discusses the case of a multiple input, single output linear system, with no dynamics and quadratic cost. Extensions for multiple output, and for finite impulse response systems are straightforward. The simple problem discussed there has, nevertheless, a number of industrial applications in process control. Heuristics that directly implement the trade-off between the estimation of system parameters and the optimization of system outputs were researched and systematized. In particular, we investigate policies that involve the on-line solution of a convex program that implements a trade-off between predictive control and experiment design. The exact solution to the problem is given by a dynamic program. However, even for relatively small problems, there is no practicable numerical solution method for this program. In [LB99], we introduce an approximation which results in a semidefinite program. Numerical experiments are encouraging, showing that the convex approximation results in an effective dual control policy for the class of problems addressed.

2.4 Matrix optimization for robust control

2.4.1 Computation of uncertainty models

In robust control design, there is a direct tradeoff between plant uncertainty, and achievable performance, *i.e.*, large uncertainty in the plant model leads to poor closed loop performance. Thus it is desirable to have the *least conservative uncertainty model*, which captures the expected plant variations. Our work has addressed the following problem: given a set of measurements of the frequency response of a (possibly MIMO) plant, taken over a finite set of operating conditions, find the tightest nonparametric frequency domain uncertainty model that is consistent with the data.

While there are a number of excellent books on the subject of robust control , all of these texts generally presume that the uncertainty weights and models are known a priori. There is very little discussion on how these models might be computed from real data, and even then, only the most elementary methods are presented, which can lead to very conservative designs, in our experience. Infact, it has been observed, in practice, that sophisticated robust optimal controllers can perform worse than much simpler heuristic "hand-tweaked" controllers, if the uncertainty model used for the robust optimization is too conservative. It is only in the past several years that researchers have begun to seriously address this issue.

We have shown that for two of the most widely used uncertainty models , the elementwise additive and matrix additive models, the problem of computing tight uncertainty models from frequency domain data can be reduced to solving a convex optimization problem, at each frequency, which *simultaneously* searches for the responses of both the nominal system and the uncertainty weights that give the tightest uncertainty model. Then a fit for the tightest nonparametric uncertainty model can be obtained using the new powerful frequency domain system identification tools . The result is a simple procedure for going directly from data to tight uncertainty model, making it much easier to carry out robust control in practice.

The techniques are demonstrated on a MIMO numerical example. The proposed techniques produce uncertainty descriptions that are over 50% tighter than averaging-based methods in the frequency regions of largest uncertainty. This research is reported in [HSB01].

2.4.2 Multi-objective control

We also pursued the problem of *multiobjective $\mathcal{H}_2/\mathcal{H}_\infty$ optimal controller design*. Based on a method proposed by Scherer, the problem can be formulated as an SDP and suboptimal solutions can be computed using *finite dimensional Q-parameterization*. The objective value of the suboptimal Q's converges to the true optimum as the dimension of Q is increased. This research has resulted in the conference paper [HHB98b].

2.5 Optimization of control and communication resources

2.5.1 Joint optimization of communication resources and linear systems

Traditionally, the problem of allocating communication channel resources has been treated separately from the problem of optimizing the performance of linear systems. In this research we explore the notion of doing both *allocation and optimization simultaneously* . We consider a linear system, such as a controller or estimator, in which several signals are transmitted over communication channels with bit rate limitations. We focus on finding the allocation of communication resources such as transmission powers, bandwidths, or time-slot fractions, that yields optimal system performance.

Assuming conventional uniform quantization and a standard white-noise model for quantization errors , we consider two specific problems. In the first, we assume that the linear system is fixed and address the problem of allocating communication resources to optimize system performance. We observe that this problem is often convex (at least, when we ignore the constraint that individual quantizers have an integral number of bits), hence readily solved. We describe a general dual decomposition method for solving these problems that exploits the special structure often found in network resource allocation problems. This method reduces to the standard waterfilling techniques used in problems with only one coupling constraint. We briefly describe how the integer bit constraints can be handled, and give a bound on how suboptimal these heuristics can be.

The second problem we consider is that of jointly allocating communication resources and designing the linear system in order to optimize system performance.

This problem is in general not convex, but can be solved heuristically in a way that exploits the special structure of the communication resource allocation problems, and appears to work well in practice.

We demonstrate these ideas and methods on two numerical examples. In the first, we consider a networked estimator in which sensors transmit measurements over a multiple access channel, and we optimize bandwidth, power allocation, and bit rates to the sensors. In the second example, we consider a networked LQG controller, in which the sensor signals are transmitted over a multiple access channel and the actuator signals are transmitted over a broadcast channel. The sensor and actuator channels have separate power limits, but share a common bandwidth constraint. Here we allocate power and bandwidth to each actuator and sensor channel, as well as the total bandwidth available to the sensors and actuators, and in addition optimize the controller itself. This research is reported in [XJH+01].

2.6 Miscellaneous work

Finally, we also finished work that had been initiated in previous years through AFOSR contracts [BCH98, HB98c, HBL97, HBL01, HBL98b, HBL98a, VBW98, VBE98, HB95, HB96, CRA+97, CRA+98, MTHBC98, LVB97, WB96, VB98, WBV98a, WBV98b, WBV98a, Boy99, HBVL97, HB98a, HB98b, HHB98a, HB97]. This work, in areas such as filter design, GPS, analog and digital circuit design, and model predictive control, are all related through the basic idea of exploiting nonlinear convex programming and Lyapunov functions.

3 Personnel supported

Stephen P. Boyd, César A. R. Crusius (research assistant), Miguel S. Lobo (research assistant).

4 Major Presentations/Honors/Awards

- *2001 ACC Plenary Panel: "From University to Wall Street"*, Washington DC.
- *Hurwitz Memorial Lecture,* ETH, Zurich.
- *Distinguished Lecturer,* Dutch Institute of Systems & Control, Eindhoven.
- *Birck Distinguished Lecture,* Purdue University.
- *MIT Distinguished Speaker Series in High Performance Computation for Engineered Systems (HPCES).*
- *O. Hugo Schuck Best Paper Award,* for 1998 ACC paper, "Multiobjective H_2/H_∞-Optimal Control via Finite Dimensional Q-Parametrization and Linear Matrix Inequalities," by H. Hindi, B. Hassibi, and S. Boyd.
- *Best student paper award,* 1999 ACC, with (student) co-author Miguel Lobo, "Policies for Simultaneous Estimation and Optimization."
- Plenary lecture, *1999 IEEE Conference on Computer-Aided Control System Design*, Kona, Hawaii.
- Plenary lecture, *7th Mediterranean Conference on Control and Automation*, Haifa, Israel.
- Election as *IEEE Fellow*, Control Systems Society. Citation: "For contributions to the design and analysis of control systems using convex optimization based CAD tools."
- Plenary lecture, Michigan Institute of Multidisciplinary Mathematics (MIMM II), University of Michigan.
- Plenary lecture, Congres d'Analyse Numerique (CANum), Ax-les-Thermes, France.
- Many lectures at Universities in the US (e.g., Univ. Minn., UCLA, etc) and in Sweden during June and July 1999 (KTH, Lund, Linkoping).

5 Publications

See references section.

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